

2.5 Reason Using Properties from Algebra



Before

You used deductive reasoning to form logical arguments.

Now

You will use algebraic properties in logical arguments too.

Why

So you can apply a heart rate formula, as in Example 3.

Key Vocabulary

- **equation**, p. 875
- **solve an equation**, p. 875

When you *solve an equation*, you use properties of real numbers. Segment lengths and angle measures are real numbers, so you can also use these properties to write logical arguments about geometric figures.

KEY CONCEPT

For Your Notebook

Algebraic Properties of Equality

Let a , b , and c be real numbers.

Addition Property If $a = b$, then $a + c = b + c$.

Subtraction Property If $a = b$, then $a - c = b - c$.

Multiplication Property If $a = b$, then $ac = bc$.

Division Property If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Substitution Property If $a = b$, then a can be substituted for b in any equation or expression.

EXAMPLE 1 Write reasons for each step

Solve $2x + 5 = 20 - 3x$. Write a reason for each step.

Equation	Explanation	Reason
$2x + 5 = 20 - 3x$	Write original equation.	Given
$2x + 5 + 3x = 20 - 3x + 3x$	Add $3x$ to each side.	Addition Property of Equality
$5x + 5 = 20$	Combine like terms.	Simplify.
$5x = 15$	Subtract 5 from each side.	Subtraction Property of Equality
$x = 3$	Divide each side by 5.	Division Property of Equality

► The value of x is 3.

Distributive Property

$$a(b + c) = ab + ac, \text{ where } a, b, \text{ and } c \text{ are real numbers.}$$
EXAMPLE 2 Use the Distributive Property

Solve $-4(11x + 2) = 80$. Write a reason for each step.

Solution

Equation	Explanation	Reason
$-4(11x + 2) = 80$	Write original equation.	Given
$-44x - 8 = 80$	Multiply.	Distributive Property
$-44x = 88$	Add 8 to each side.	Addition Property of Equality
$x = -2$	Divide each side by -44 .	Division Property of Equality

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EXAMPLE 3 Use properties in the real world

HEART RATE When you exercise, your target heart rate should be between 50% to 70% of your maximum heart rate. Your target heart rate r at 70% can be determined by the formula $r = 0.70(220 - a)$ where a represents your age in years. Solve the formula for a .

Solution

Equation	Explanation	Reason
$r = 0.70(220 - a)$	Write original equation.	Given
$r = 154 - 0.70a$	Multiply.	Distributive Property
$r - 154 = -0.70a$	Subtract 154 from each side.	Subtraction Property of Equality
$\frac{r - 154}{-0.70} = a$	Divide each side by -0.70 .	Division Property of Equality

**GUIDED PRACTICE** for Examples 1, 2, and 3

In Exercises 1 and 2, solve the equation and write a reason for each step.

- $4x + 9 = -3x + 2$
- $14x + 3(7 - x) = -1$
- Solve the formula $A = \frac{1}{2}bh$ for b .

PROPERTIES The following properties of equality are true for all real numbers. Segment lengths and angle measures are real numbers, so these properties of equality are true for segment lengths and angle measures.

KEY CONCEPT

For Your Notebook

Reflexive Property of Equality

Real Numbers For any real number a , $a = a$.

Segment Length For any segment \overline{AB} , $AB = AB$.

Angle Measure For any angle $\angle A$, $m\angle A = m\angle A$.

Symmetric Property of Equality

Real Numbers For any real numbers a and b , if $a = b$, then $b = a$.

Segment Length For any segments \overline{AB} and \overline{CD} , if $AB = CD$, then $CD = AB$.

Angle Measure For any angles $\angle A$ and $\angle B$, if $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Transitive Property of Equality

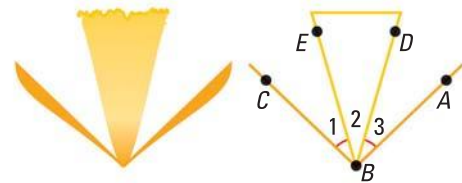
Real Numbers For any real numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.

Segment Length For any segments \overline{AB} , \overline{CD} , and \overline{EF} , if $AB = CD$ and $CD = EF$, then $AB = EF$.

Angle Measure For any angles $\angle A$, $\angle B$, and $\angle C$, if $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

EXAMPLE 4 Use properties of equality

LOGO You are designing a logo to sell daffodils. Use the information given. Determine whether $m\angle EBA = m\angle DBC$.

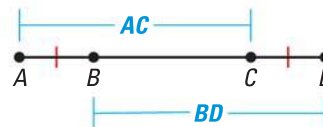


Solution

Equation	Explanation	Reason
$m\angle 1 = m\angle 3$	Marked in diagram.	Given
$m\angle EBA = m\angle 3 + m\angle 2$	Add measures of adjacent angles.	Angle Addition Postulate
$m\angle EBA = m\angle 1 + m\angle 2$	Substitute $m\angle 1$ for $m\angle 3$.	Substitution Property of Equality
$m\angle 1 + m\angle 2 = m\angle DBC$	Add measures of adjacent angles.	Angle Addition Postulate
$m\angle EBA = m\angle DBC$	Both measures are equal to the sum of $m\angle 1 + m\angle 2$.	Transitive Property of Equality

EXAMPLE 5 Use properties of equality

In the diagram, $AB = CD$. Show that $AC = BD$.



Solution

Equation	Explanation	Reason
$AB = CD$	Marked in diagram.	Given
$AC = AB + BC$	Add lengths of adjacent segments.	Segment Addition Postulate
$BD = BC + CD$	Add lengths of adjacent segments.	Segment Addition Postulate
$AB + BC = CD + BC$	Add BC to each side of $AB = CD$.	Addition Property of Equality
$AC = BD$	Substitute AC for $AB + BC$ and BD for $BC + CD$.	Substitution Property of Equality



GUIDED PRACTICE for Examples 4 and 5

Name the property of equality the statement illustrates.

- If $m\angle 6 = m\angle 7$, then $m\angle 7 = m\angle 6$.
- If $JK = KL$ and $KL = 12$, then $JK = 12$.
- $m\angle W = m\angle W$

2.5 EXERCISES

HOMEWORK KEY

- = WORKED-OUT SOLUTIONS on p. WS1 for Exs. 9, 21, and 31
- = STANDARDIZED TEST PRACTICE Exs. 2, 5, 27, and 35
- = MULTIPLE REPRESENTATIONS Ex. 36

SKILL PRACTICE

1. **VOCABULARY** The following statement is true because of what property? The measure of an angle is equal to itself.

2. **★ WRITING** Explain how to check the answer to Example 3 on page 106.

WRITING REASONS Copy the logical argument. Write a reason for each step.

3.	$3x - 12 = 7x + 8$	Given	4.	$5(x - 1) = 4x + 13$	Given
	$-4x - 12 = 8$?		$5x - 5 = 4x + 13$?
	$-4x = 20$?		$x - 5 = 13$?
	$x = -5$?		$x = 18$?

EXAMPLES 1 and 2

on pp. 105–106 for Exs. 3–14

5. ★ **MULTIPLE CHOICE** Name the property of equality the statement illustrates: If $XY = AB$ and $AB = GH$, then $XY = GH$.

(A) Substitution (B) Reflexive (C) Symmetric (D) Transitive

WRITING REASONS Solve the equation. Write a reason for each step.

6. $5x - 10 = -40$ 7. $4x + 9 = 16 - 3x$ 8. $5(3x - 20) = -10$
 9. $3(2x + 11) = 9$ 10. $2(-x - 5) = 12$ 11. $44 - 2(3x + 4) = -18x$
 12. $4(5x - 9) = -2(x + 7)$ 13. $2x - 15 - x = 21 + 10x$ 14. $3(7x - 9) - 19x = -15$

EXAMPLE 3

on p. 106
for Exs. 15–20

xy ALGEBRA Solve the equation for y . Write a reason for each step.

15. $5x + y = 18$ 16. $-4x + 2y = 8$ 17. $12 - 3y = 30x$
 18. $3x + 9y = -7$ 19. $2y + 0.5x = 16$ 20. $\frac{1}{2}x - \frac{3}{4}y = -2$

EXAMPLES 4 and 5

on pp. 107–108
for Exs. 21–25

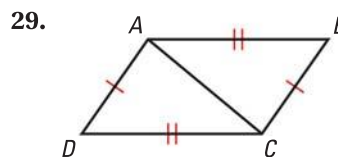
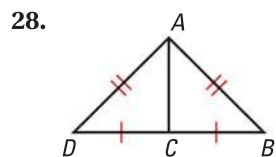
COMPLETING STATEMENTS In Exercises 21–25, use the property to copy and complete the statement.

21. Substitution Property of Equality: If $AB = 20$, then $AB + CD = \underline{\quad? \quad}$.
 22. Symmetric Property of Equality: If $m\angle 1 = m\angle 2$, then $\underline{\quad? \quad}$.
 23. Addition Property of Equality: If $AB = CD$, then $\underline{\quad? \quad} + EF = \underline{\quad? \quad} + EF$.
 24. Distributive Property: If $5(x + 8) = 2$, then $\underline{\quad? \quad}x + \underline{\quad? \quad} = 2$.
 25. Transitive Property of Equality: If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $\underline{\quad? \quad}$.
 26. **ERROR ANALYSIS** Describe and correct the error in solving the equation for x .

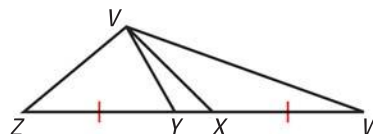
$7x = x + 24$	Given	
$8x = 24$	Addition Property of Equality	
$x = 3$	Division Property of Equality	

27. ★ **OPEN-ENDED MATH** Write examples from your everyday life that could help you remember the *Reflexive*, *Symmetric*, and *Transitive* Properties of Equality.

PERIMETER In Exercises 28 and 29, show that the perimeter of triangle ABC is equal to the perimeter of triangle ADC .



30. **CHALLENGE** In the figure at the right, $\overline{ZY} \cong \overline{XW}$, $ZX = 5x + 17$, $YW = 10 - 2x$, and $YX = 3$. Find ZY and XW .



PROBLEM SOLVING

EXAMPLE 3

on p. 106
for Exs. 31–32

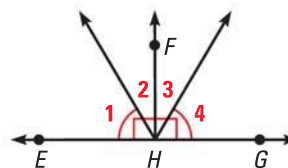
- 31. PERIMETER** The formula for the perimeter P of a rectangle is $P = 2\ell + 2w$ where ℓ is the length and w is the width. Solve the formula for ℓ and write a reason for each step. Then find the length of a rectangular lawn whose perimeter is 55 meters and whose width is 11 meters.

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- 32. AREA** The formula for the area A of a triangle is $A = \frac{1}{2}bh$ where b is the base and h is the height. Solve the formula for h and write a reason for each step. Then find the height of a triangle whose area is 1768 square inches and whose base is 52 inches.

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- 33. PROPERTIES OF EQUALITY** Copy and complete the table to show $m\angle 2 = m\angle 3$.



Equation	Explanation	Reason
$m\angle 1 = m\angle 4, m\angle EHF = 90^\circ,$ $m\angle GHF = 90^\circ$?	Given
$m\angle EHF = m\angle GHF$?	Substitution Property of Equality
$m\angle EHF = m\angle 1 + m\angle 2$ $m\angle GHF = m\angle 3 + m\angle 4$	Add measures of adjacent angles.	?
$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	Write expressions equal to the angle measures.	?
?	Substitute $m\angle 1$ for $m\angle 4$.	?
$m\angle 2 = m\angle 3$?	Subtraction Property of Equality

- 34. MULTI-STEP PROBLEM** Points $A, B, C,$ and D represent stops, in order, along a subway route. The distance between Stops A and C is the same as the distance between Stops B and D .

- a. Draw a diagram to represent the situation.
- b. Use the Segment Addition Postulate to show that the distance between Stops A and B is the same as the distance between Stops C and D .
- c. *Justify* part (b) using the Properties of Equality.

EXAMPLE 4

on p. 107
for Ex. 35

- 35. ★ SHORT RESPONSE** A flashlight beam is reflected off a mirror lying flat on the ground. Use the information given below to find $m\angle 2$.

$$m\angle 1 + m\angle 2 + m\angle 3 = 180^\circ$$

$$m\angle 1 + m\angle 2 = 148^\circ$$

$$m\angle 1 = m\angle 3$$



36. **MULTIPLE REPRESENTATIONS** The formula to convert a temperature in degrees Fahrenheit ($^{\circ}\text{F}$) to degrees Celsius ($^{\circ}\text{C}$) is $C = \frac{5}{9}(F - 32)$.
- Writing an Equation** Solve the formula for F . Write a reason for each step.
 - Making a Table** Make a table that shows the conversion to Fahrenheit for each temperature: 0°C , 20°C , 32°C , and 41°C .
 - Drawing a Graph** Use your table to graph the temperature in degrees Celsius ($^{\circ}\text{C}$) as a function of the temperature in degrees Fahrenheit ($^{\circ}\text{F}$). Is this a linear function?

CHALLENGE In Exercises 37 and 38, decide whether the relationship is reflexive, symmetric, or transitive.

37. **Group:** two employees in a grocery store
Relationship: “worked the same hours as”
Example: Yen worked the same hours as Jim.
38. **Group:** negative numbers on a number line
Relationship: “is less than”
Example: -4 is less than -1 .

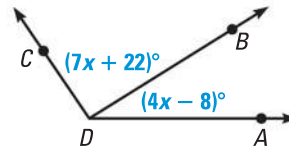
MIXED REVIEW

PREVIEW

Prepare for Lesson 2.6 in Exs. 39–40.

In the diagram, $m\angle ADC = 124^{\circ}$. (p. 24)

- Find $m\angle ADB$.
- Find $m\angle BDC$.



- Find a counterexample to show the conjecture is false.

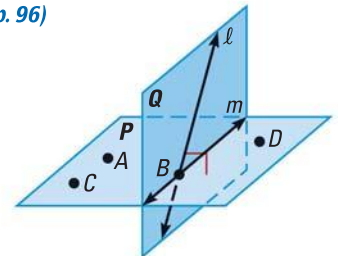
Conjecture All polygons have five sides. (p. 72)

- Select the word(s) that make(s) the conclusion true. If $m\angle X = m\angle Y$ and $m\angle Y = m\angle Z$, then $m\angle X$ (is, may be, or is not) equal to $m\angle Z$. (p. 87)

QUIZ for Lessons 2.4–2.5

Use the diagram to determine if the statement is true or false. (p. 96)

- Points B , C , and D are coplanar.
- Point A is on line ℓ .
- Plane P and plane Q are perpendicular.



Solve the equation. Write a reason for each step. (p. 105)

- $x + 20 = 35$
- $5x - 14 = 16 + 3x$

Use the property to copy and complete the statement. (p. 105)

- Subtraction Property of Equality: If $AB = CD$, then $\underline{\quad} - EF = \underline{\quad} - EF$.
- Transitive Property of Equality: If $a = b$ and $b = c$, then $\underline{\quad} = \underline{\quad}$.